Complex-based controller for a three-phase inverter with an LCL filter connected to unbalanced grids

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***Abstract*—A new controller for a grid connected inverter with an LCL filter is proposed in this paper. The system is described by its complex representation and the controller is designed using the complex root locus method. The complex representation allows a considerable reduction in the order of the system, simplifying the design task and making it possible to use of advanced techniques such as the complex root locus. The new complex controller adds an extra degree of freedom that makes it possible to move the poles of the systems, and improve stability and speed of response compared to the conventional controls. The paper include a detailed discussion of the effect of the gains of the controller on the root locus. The proposal is validated with simulation and experimental results.**

***Index Terms*—Complex control, LCL filter, three-phase inverter, root-locus rules.**

I. INTRODUCTION

ELECTRIC-power applications (especially the renewable ones such as photovoltaic, wind power,...) require high performance for injecting current to the power grid [1]. The use of power converters with an LCL filter has increased in the last years for grid connection operations due to the better attenuation of the current harmonics and switching frequency in comparison with the traditional L filters [2]. On the other hand, power converters with LCL filters require more advanced control strategies, because the system order increases and a resonance peak appears that may cause stability problems in the presence of parameters variations [1].

Several control techniques have been applied to three-phase power converters with an LCL filter. The proposed methods include classical linear techniques such as PI controllers [3], phase compensators [4], resonant controllers [5], linear Lyapunov tools [6] a state-feedback approach [7], linear quadratic regulators [8], and LQG state-feedback controller with resonators [9]. Other examples include Model Predictive Control [10], the use of observers [11], and pseudo-derivative-feedback control method [12]. One of the most common approach is the Weighted Average Current Control strategy (WACC) [13]. The WACC methodology has been recently studied in [14] and [15]. Also, nonlinear techniques have been used for LCL inverters. Examples include Sliding Mode Control [16] [17], nonlinear feedback and Poincaré analysis [18].

On the other hand, some papers propose controller designs for LCL inverters connected to unbalanced grids. Generally, the schemes consist in controlling in parallel the positive and negative sequences. For example, [19] presents a modified cascaded boundary-deadbeat control law for a four-wire inverter, [20] studies the current injection without the need of phase-locked loop (PLL) calculation, or [21] that also considers the operation under grid faults and the fault ride through capability. Other alternatives can be found in [22] that mixes robust predictive control and sliding modes, in [23], [24] where resonant controllers are considered, and [25] that uses a pole placement technique.

In most cases, tuning the controller gains proceeds by trial and error, without a comprehensive understanding of the resulting dynamics. The main reason for this shortcoming is that LCL inverters are described by a 6th order model in the dq or $\alpha\beta$ coordinates, so that the analysis of the closed-loop system is complicated. The objective of this paper is to improve the design methods and their analysis based on a complex description of the inverter.

The analysis of systems described by transfer functions with complex coefficients was first proposed in [26], and later used in [27] for control and analysis of induction machines. Then, the approach was applied to current regulators in [28] and [29]. Some time later, an overview of applications to three-phase systems with complex transfer functions was published in [30]. Recently, [31] initiated a new effort applying stability analysis tools and design techniques in the complex domain, that was extended in [32][33]. The main advantage is the reduction of the order of the system that facilitates the analysis and the synthesis of the controllers [34]. Recently, some frequency-domain results were revisited in [35]. Additionally, filters based on complex coefficients have been proposed for PLLs and synchronization techniques [36].

The root locus method was extended to the complex domain in [37], and its application to three-phase electrical systems was proposed in [14]. As a result of the simplification, it is possible to develop control schemes for which tuning rules can be provided for the control gains. The main contribution of this paper is the use of new control design tools for a power inverter using a complex transfer function description and the evaluation of the resulting feedback system on a real platform. Thanks to the use of complex-based techniques, the design and the analysis are simplified, converting the multiple-input multiple-output (MIMO) problem into a single-input single-output (SISO) problem. Compared to [14], the

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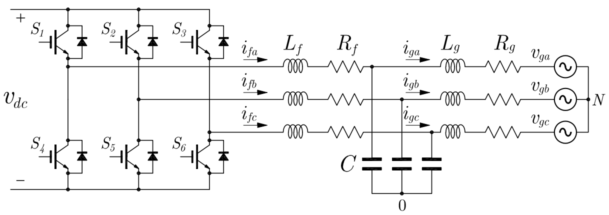

Fig. 1. Electrical scheme of a three-phase grid inverter connected with an LCL filter.

novelty of the paper consists in: i) a detailed gain design/tuning method, ii) a study of the effect of parameter variations, iii) a control scheme considering the case of unbalanced grids, iv) a quantitative analysis of the effect of delays on the stability, and v) the implementation on a real platform, yielding experimental results.

The paper is organized as follows. Section II details the modelling procedure to obtain the transfer function with complex coefficients. The control design and the gain tuning discussion are presented in Section III. Then, some practical issues are considered in Section IV, including parametric variations, unbalanced grids and the effect of feedback delays. Simulation and experimental results are presented in Section V and, finally, conclusions are stated in Section VI.

II. Three-phase LCL inverter dynamical model

A. Dynamical model

A three-phase grid-connected inverter with an LCL filter is shown in Figure 1. The dynamics of this system are described by,

$$\begin{aligned} L_f\frac{\mathrm{d}\boldsymbol{i}_f^{abc}}{\mathrm{d}t} &= -R_f\boldsymbol{i}_f^{abc} - \boldsymbol{M}\boldsymbol{v}_C^{abc} + \frac{1}{2}v_{dc}\boldsymbol{M}\boldsymbol{u}^{abc}, && (1)\\ L_g\frac{\mathrm{d}\boldsymbol{i}_g^{abc}}{\mathrm{d}t} &= -R_g\boldsymbol{i}_g^{abc} + \boldsymbol{M}\boldsymbol{v}_C^{abc} - \boldsymbol{N}\boldsymbol{v}_g^{l}, && (2)\\ C\frac{\mathrm{d}\boldsymbol{v}_C^{abc}}{\mathrm{d}t} &= \boldsymbol{i}_f^{abc} - \boldsymbol{i}_g^{abc}, && (3)\end{aligned}$$

where $\boldsymbol{i}_f^{abc} = (i_{fa}, i_{fb}, i_{fc})^T, \boldsymbol{i}_g^{abc} = (i_{ga}, i_{gb}, i_{gc})^T$ are the inverter-side and the grid-side currents of the LCL filters, respectively, $\boldsymbol{v}_C^{abc} = (v_{Ca}, v_{Cb}, v_{Cc})^T$ are the voltages in the filter capacitors, with capacitance C, and $\boldsymbol{v}_g^l = (v_g^{ab}, v_g^{bc})^T$ are the measured line-to-line voltages. The resistances R_f and R_g represent the losses in the filter inductors L_f and L_g. The control signals, $\boldsymbol{u}^{abc} = (u_a, u_b, u_c)^T$, take the discrete values $u_k \in \{-1, 1\}$, for $k = a, b, c$. The matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ are,

$$\boldsymbol{M} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}, \quad \boldsymbol{N} = \frac{1}{3}\begin{pmatrix} 2 & 1\\ -1 & 1\\ -1 & -2\end{pmatrix}.$$

B. Complex representation of the LCL inverter

It is well-known that a set of three-phase sinusoidal functions (electrical variables) $\boldsymbol{f}_{abc} = (f_a(t), f_b(t), f_c(t))$, with the same frequency, ω_g, can be expressed as a sum of three sets of symmetrical variables,

$$\boldsymbol{f}_{abc} = \boldsymbol{f}_{abc}^0 + \boldsymbol{f}_{abc}^+ + \boldsymbol{f}_{abc}^-, \tag{4}$$

where in this case, because the absence of neutral connection, the zero-sequence is $\boldsymbol{f}_{abc}^0 = 0$. Similarly to [14], the complex representation of (4), is obtained applying the abc to $\alpha\beta$ transformation,

$$f_{\alpha\beta} = f_\alpha + jf_\beta = \boldsymbol{T}\boldsymbol{f}_{abc}, \tag{5}$$

with[1] $\boldsymbol{T} = \sqrt{\frac{2}{3}}(1, e^{j\frac{2\pi}{3}}, e^{-j\frac{2\pi}{3}})$, and defining,

$$\begin{aligned} f_{dq}^+ &= f_d + jf_q = e^{-j\theta}f_{\alpha\beta}, && (6)\\ f_{dq}^- &= f_d + jf_q = e^{j\theta}f_{\alpha\beta}, && (7)\end{aligned}$$

where $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \hat{\omega}_g$ and $\hat{\omega}_g$ is the estimated grid frequency.

Consequently, the dynamics (1)-(3) can be decomposed in two subsystems, for the positive (or forward) sequence,

$$\begin{aligned} L_f\frac{\mathrm{d}i_f^+}{\mathrm{d}t} &= -(R_f + j\omega_g L_f)i_f^+ - v_c^+ + v_{dc}u^+, && (8)\\ L_g\frac{\mathrm{d}i_g^+}{\mathrm{d}t} &= -(R_g + j\omega_g L_g)i_g^+ + v_c^+ - v_g^+, && (9)\\ C\frac{\mathrm{d}v_c^+}{\mathrm{d}t} &= i_f^+ - i_g^+ - j\omega_g C v_c^+, && (10)\end{aligned}$$

and the negative (or backwards) sequence,

$$\begin{aligned} L_f\frac{\mathrm{d}i_f^-}{\mathrm{d}t} &= -(R_f - j\omega_g L_f)i_f^- - v_c^- + v_{dc}u^-, && (11)\\ L_g\frac{\mathrm{d}i_g^-}{\mathrm{d}t} &= -(R_g - j\omega_g L_g)i_g^- + v_c^- - v_g^-, && (12)\\ C\frac{\mathrm{d}v_c^-}{\mathrm{d}t} &= i_f^- - i_g^- + j\omega_g C v_c^-, && (13)\end{aligned}$$

where the state variables (omitting subindices $+$ and $-$) are $i_f = i_{fd} + ji_{fq}$, $i_g = i_{gd} + ji_{gq}$, $v_c = v_{cd} + jv_{cq}$, the grid voltages $v_g = v_{gd} + jv_{gq}$, and the control inputs $u = u_d + ju_q$. As the dynamics (8)-(10) and (11)-(13) are similar, from now on, the transfer function model and the control design is only detailed for the positive sequence[2]. The control design consists in separating the problem into two regulation schemes.

The open loop dynamics (8)-(10) can be written as,

$$\begin{pmatrix} N_f(s) & 0 & 1\\ 0 & N_g(s) & -1\\ -1 & 1 & N_c(s)\end{pmatrix}\begin{pmatrix} i_f\\ i_g\\ v_c\end{pmatrix} = \begin{pmatrix} v_{dc}u\\ v_g\\ 0\end{pmatrix},$$

where,

$$\begin{aligned} N_f(s) &= (s + j\omega_g)L_f + R_f, && (14)\\ N_g(s) &= (s + j\omega_g)L_g + R_g, && (15)\\ N_c(s) &= (s + j\omega_g)C. && (16)\end{aligned}$$

Finally, the system has a complex transfer function,

$$i_g = \frac{v_{dc}}{D_{OL}(s)}u + \frac{1 + N_f(s)N_c(s)}{D_{OL}(s)}v_g, \tag{17}$$

where,

$$D_{OL}(s) = N_f(s) + N_g(s) + N_f(s)N_g(s)N_c(s),$$

[1] Notice that we choose in (5) a power-preserving transformation.

[2] For simplicity, in the derived transfer function and control design, the subindices $+$ are omitted.

or, splitting into the real and imaginary parts,

$$D_{OL}(s) = N_r(s) + jN_i(s), \tag{18}$$

with,

$$\begin{aligned} N_r(s) =& CL_fL_gs^3 + C(L_fR_g + L_gR_f)s^2 \\ &+ (CR_fR_g - 3CL_fL_g\omega_g^2 + L_f + L_g)s \\ &- \omega_g^2C(L_fR_g + L_gR_f) + R_f + R_g, \end{aligned} \tag{19}$$

$$\begin{aligned} N_i(s) =& 3\omega_gCL_fL_gs^2 + 2C(L_fR_g\omega_g + L_gR_f\omega_g)s \\ &- \omega_g^3CL_fL_g + \omega_gCR_fR_g + \omega_g(L_f + L_g). \end{aligned} \tag{20}$$

III. Control design

In this section, the controller proposed in [14] is reviewed and a detailed discussion on the gain tuning procedure for a real application is proposed.

From (17), and treating v_g as a disturbance ($v_g = 0$), the complex transfer function of the system from the input u to the output i_g is,

$$i_g = \frac{v_{dc}}{D_{OL}(s)}u. \tag{21}$$

A classical approach for controlling the LCL three-phase inverter is to decouple the d and q components [13]. This approach is helpful for tuning the PI gain parameters and implies the same closed-loop behavior for both dq components. The decoupling is obtained with a feedforward term cancelling all the cross-terms that, in the complex representation are found in the imaginary part of $D_{OL}(s)$, *i.e.,* $N_i(s)$. Based on this observation, the controller proposed in [14] consists of a conventional PI current controller plus a feedback on i_f,

$$u = j\frac{N_i(s)}{v_{dc}}i_g - k_fi_f + k_P\left(1 + \frac{1}{T_is}\right)(i_g^{\text{ref}} - i_g), \tag{22}$$

where, k_P and T_i are the control design (real) parameters and i_g^{ref} is the grid-side dq-current reference. In (22) k_f is a complex number which adds an extra degree of freedom to place the poles.

Inserting (22) in (21), we get,

$$i_g = \frac{k_Pv_{dc}\left(s + \frac{1}{T_i}\right)}{D_{CL}(s)}i_g^{\text{ref}},$$

with,

$$D_{CL}(s) = sN_r(s) + sv_{dc}k_f(N_g(s)N_c(s)+1) + k_Pv_{dc}\left(s + \frac{1}{T_i}\right).$$

Remark 1: The conventional PI current controller is recovered with $k_f = 0$,

$$u = j\frac{N_i(s)}{v_{dc}}i_g + k_P\left(1 + \frac{1}{T_is}\right)(i_g^{\text{ref}} - i_g). \tag{23}$$

Figure 2 shows the block diagram of the proposed controller. It mainly consists of a PI controller plus two additional feedback terms on i_g and i_f, that is similar to a classical state feedback scheme, see examples in [38], [24] and [9]. The current variables (in a complex description), i_g and i_f, are obtained thanks to the abc/dq transformations corresponding to (5) and (6)-(7).

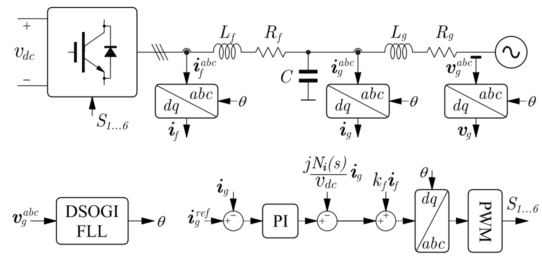

Fig. 2. Block diagram of the proposed controller.

Remark 2: The term $j\frac{N_i(s)}{v_{dc}}$ in (22) is non-causal. For implementation purposes, $N_i(s)$ can be approximated by the causal operator,

$$N_i(s) \approx \frac{a_2s^2 + a_1s + a_0}{\epsilon_2s^2 + \epsilon_1s + 1},$$

where ϵ_2, ϵ_1 are small values, see [39] for its digital implementation. For the experimental plant used in Section V, as the numerical coefficients of (20) are $a_2 = 3.239 \cdot 10^{-9}, a_1 = 1.036 \cdot 10^{-6}$ and $a_0 = 0.589$, $N_i(s)$ has been approximated by a static gain, $N_i(s) \approx a_0$.

A. Root locus analysis

First, the influence of the control parameters on the root locus is analyzed. In particular, the case of the LCL inverter described in Section V is studied.

Figure 3 shows the root locus when $k_f = 0$ for varying k_P and for different values of T_i. The open loop poles are at the origin (pole p_1) and on the negative real axis (pole p_2), with two more complex poles having negative real part (poles p_3 and p_4). Note that the poles p_3-p_4 are close to the imaginary axis and, to maintain the stability of the system, the gain k_P must be sufficiently small. Moreover, following Rule 4 of the complex root locus method [37], a breakaway point exists for $T_i \leq 0.00468 = T_i^{\text{bk}}$, that can be viewed in the detail of the complex root locus in Figure 3. Overall, the convergence speed is determined by the pole p_1 on the imaginary axis starting at the origin, but when raising k_P to increase the convergence speed, the complex poles p_3-p_4 become unstable. This implies that only slow time responses can be achieved using the conventional PI structure (23).

Figure 4 compares the root locus for $k_f = 0$ and $k_f \neq 0$. The pole at the origin, p_1, remains there, the pole p_2 move out from the real axis and to the left, and poles p_3-p_4 move to the left. For varying gain, p_1 goes to the zero at $-\frac{1}{T_i}$, while the p_2 goes to infinity asymptotically on the negative real axis. The two complex poles p_3-p_4 still cross to the right-half plane before reaching asymptotes at $\pm 60°$ (not shown). The main advantage is that the feedback on i_f moves the open-loop poles p_3-p_4 to a location farther in the left-half plane than for the conventional control law ($k_f = 0$).

Although the complex gain introduces an additional coupling between the d and q axes, it can actually be used to achieve better. In Figure 5, the starting points (open-loop poles) of the root locus are shown for different values of k_f. In blue, the variation in the location of the starting points

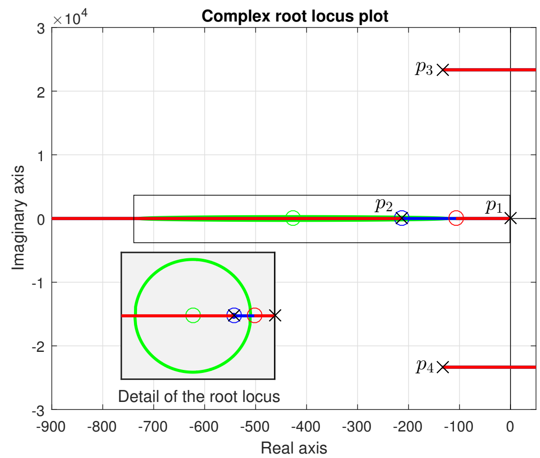

Fig. 3. Complex root locus plot of (23) with $k_f = 0$ for different values of T_i, $T_i = 0.5T_i^{\text{bk}}$ in green), $T_i = T_i^{\text{bk}}$ in blue, and $T_i = 2T_i^{\text{bk}}$ in red, where $T_i^{\text{bk}} = 0.00468$.

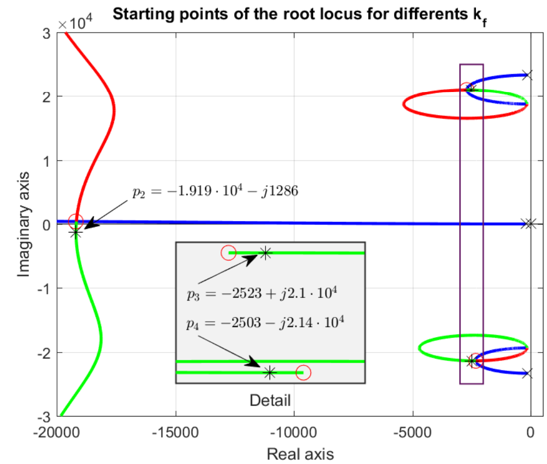

Fig. 5. Starting points ($k_P = 0$) of the root locus for the proposed controller in (22). ($\times$) poles are for $k_f = 0$, starting points for real k_f values are in blue, ($\circ$) poles are the farthest points from the real axis for a real k_f ($k_f = 0.0989$), the red and green plots are the roots for $k_f = 0.0989 + jk_{fI}$, $k_{fI} < 0$ and $k_{fI} > 0$, respectively. ($*$) poles are for $k_f = 0.0989 + j0.007$.

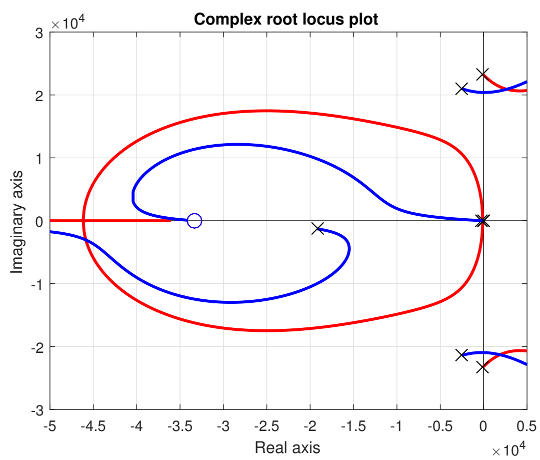

Fig. 4. Complex root locus: Comparison between the proposed controller (22) with $T_i = 3 \cdot 10^{-5}$, for $k_f = 0$ in red (conventional controller) and $k_f = 0.0989 + j0.007$ in blue.

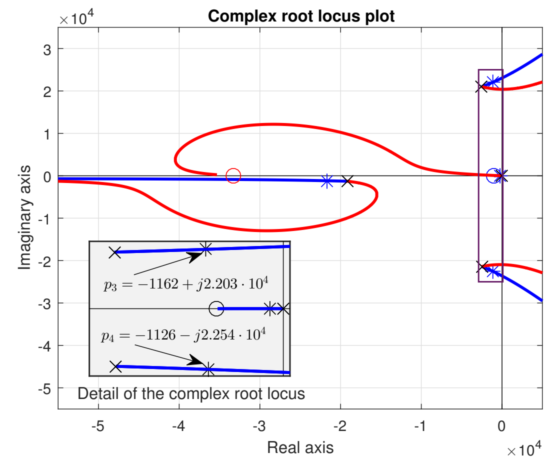

Fig. 6. Complex root locus of the complex based controller for $k_f = 0.0989 + j0.007$ and different values of T_i, $1 \cdot 10^{-3}$ (in blue) and $T_i = 3 \cdot 10^{-5}$ (in red). The selection of the poles for the specification is shown with star ($*$) marks.

when k_f is a real number. The pole p_1 remains, but the other starting points move to the left reaching a minimal value for p_3-p_4 ($\circ$ marks). The imaginary part of k_f can also be tuned (red and green lines for negative and positive imaginary values, respectively) also affecting the starting pole placement.

Finally, the effect of T_i is analyzed in Figure 6. In all cases, the root locus is similar. The main difference is where the zero $-1/T_i$ is placed, and how the pole p_1 moves to $-1/T_i$. The choice of T_i can be related to the desired performance.

B. Control gain tuning

The gain tuning consists in finding the control parameters that provides the desired performance. For the application described in Section V, the desired settling time has been set at $t_s = 20$ms (one power grid period) with minimal overshoot. As suggested by the analysis in the previous section, the pole p_1 can be used for the design (dominant pole) while placing the other poles far enough to the left.

The proposed procedure consists in four steps:

1) Selection of the k_f gain: As discussed in Figure 5, k_f is selected such that the starting points of the poles other than p_1 are further from the imaginary axis. The minimal placement of the complex starting poles is reached with $k_f = 0.0989$ ($\circ$ marks) and, using the imaginary part, for $k_f = 0.0989 + j0.007$ ($*$ marks).
2) Selection of the T_i gain: Selecting a low T_i, the zero of the controller moves away from the imaginary axis, and the pole p_1 will move faster when k_P is increasing. Additionally, from Figure 6, high T_i values imply that the branch for p_1, starting at the origin, remains close to the real axis. A trade-off setting corresponds to $T_i = 1 \cdot 10^{-3}$ (blue line in Figure 6) that gives a minimum overshoot and allows to select an appropriate k_P for the specifications.
3) Selection of the k_P gain: The gain k_P is selected such that the real part of the dominant pole, p_1, is $\sigma = -200$.

The value corresponds to $k_P = 0.025$.

4) Verification: Once the gains are obtained, the last step consists in verifying that the desired pole is dominant and, if necessary, reducing the speed of the response. In this case, the closed loop poles are places at $p_1 = -201.1 + j11.46$ (dominant pole) and $p_2 = -2.173 \cdot 10^4 - j1174$, $p_3 = -1162 + j2.203 \cdot 10^4$, $p_4 = -1126 + j2.254 \cdot 10^4$, with star $(*)$ marks in Figure 6. With a factor greater than 5 between the real part of the first pole and the real part of the other poles, no further change is found necessary.

IV. Analysis and Design Tools for Practical Issues

A. Robustness in front of inductance variations

This section analyzes the root locus in the presence of parametric variations of L_f and L_g. These variations originate from different causes, such as saturations, temperature effects, aging of the inductors, and others. Also, the influence of the grid impedance could be roughly accounted for as a variation of L_g (increasing).

Figure 7 shows how the closed loop poles move when L_f (red) and L_g (blue) change from 0.5 to 5 times the nominal values (these huge variations are considered in order to observe the effects). The poles obtained with the gains selected in the previous section are indicated with star marks, $(*)$, the closed loop poles for x0.5 the nominal value are marked with upside down triangles $(\triangledown)$, and ones for x5 the nominal value are with square marks $(\square)$.

It can be seen that, when L_g increases, the dominant pole (p_1) moves to the left but remains close to the desired value. Poles p_3-p_4 move to the left (increasing the stability), and p_2 moves to the right, but without affecting the dominance. When L_g is decreased, p_3-p_4 move to the right. Also, p_1 slightly moves to the left, increasing the speed response (see detail of the pole movement in Figure 7). A similar discussion can be done for variations of L_f. Notice that, even though poles p_2, p_3, and p_4 significantly move with variations of L_g and L_f, p_1 remains close to the nominal placement.

For the system considered in this paper, a typical variation (reduction of 10%) in L_g was tested and the following values were obtained $p_1 = -201 + j11.45$, $p_2 = -2.207 \cdot 10^4 - j1182$, $p_3 = -1021 + j2.307 \cdot 10^4, p_4 = -963.4 + j2.358 \cdot 10^4$. The dominance of p_1 with respect to p_2, p_3 and p_4 approximately remains. Thus, the desired performance for the designed controller is not significantly affected. In a case where the L_g variation compromises the performance, the tuning procedure described above should be repeated with less restrictive requirements.

B. Design for unbalanced power networks

The control scheme presented in Section III can be adapted to guarantee balanced grid currents by adding a loop for the negative current sequence [40]. The design follows the model derived in Section II, but using (11)-(13). The only difference is that Equations (14)-(16) now yield,

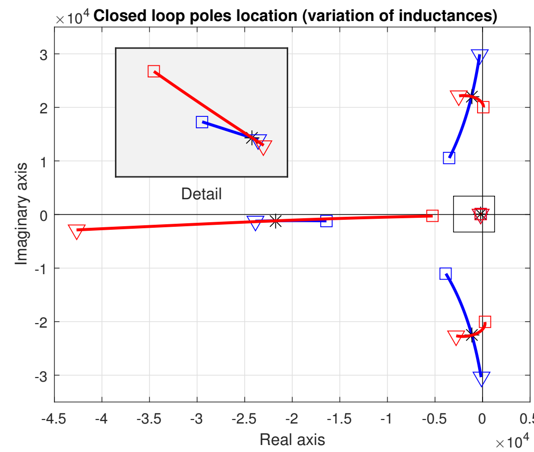

Fig. 7. Closed loop poles location of the complex based controller for a variation in L_g and L_f between x0.5 and x5. The poles for selected gains are indicated with star marks $(*)$, the closed loop poles for x0.5 the nominal value are marked with upside down triangles $(\triangledown)$, and ones for x5 the nominal value are with square marks $(\square)$.

$$N_f(s) = (s - j\omega_g)L_f + R_f,$$
$$N_g(s) = (s - j\omega_g)L_g + R_g,$$
$$N_c(s) = (s - j\omega_g)C,$$

and, consequently, (18) changes to,

$$D_{OL}(s) = N_r(s) - jN_i(s),$$

where $N_r(s), N_i(s)$ are the same as (19)-(20). Notice that, for the negative sequence, the root-locus is the conjugate of the one obtained for the positive sequence, and all the tuning procedure remains similar.

Then, the proposed controller for the negative sequence is similar to (22), with a minus in the feedforward term, *i.e.,*

$$u^- = -j\frac{N_i(s)}{v_{dc}}i_g^- - k_f^- i_f^- + k_P^- \left(1 + \frac{1}{T_i^- s}\right)(i_g^{\text{ref}-} - i_g^-). \tag{24}$$

Overall, the control scheme is the addition of the two current loops (22) and (24), as shown in Figure 8, where the DSOGI-FLL algorithm [41] is included to determine both positive and negative sequences. The DSOGI-FLL method together with the positive (or negative) sequence controller is a narrow-band system centered around the positive (or negative) grid frequency. In particular, the components at plus (or minus) two times the grid frequency are approximately removed. This makes it possible to design the feedback loops for the two components to be designed separately, with the two control signals simply added together.

The gain tuning procedure of (24) is the same as the one described in Section III. In order to decouple the dynamics of the positive and negative sequences, it is recommended to design the negative sequence response slower than the positive sequence. For the case considered in Section V, the desired settling time is set at $t_s = 200$ms, and can be obtained with, $k_f^- = 0.0989 + j0.007, T_i^- = 10^{-3}$ and $k_P^- = 0.002$.

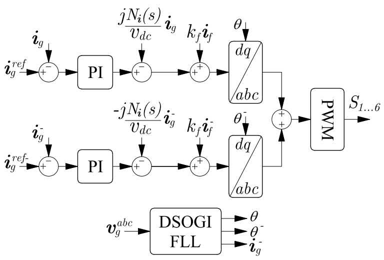

Fig. 8. Block diagram of the complete controller with the positive and negative sequence loops.

C. Robustness in the presence of time delays

Delays appear in the feedback loop due to the filter on the currents and to the sampled-data system. The effect of these delays on the stability can be studied by including the dynamics of the filter and a Padé approximation of the delay in the procedure of Section III. An interesting option provided by the complex analysis is to compute the delay margin of the feedback system based on the phase of the complex loop transfer function at the crossover frequency (or frequencies) [30]. The computation of the delay margin originates from the Nyquist criterion and can interpreted from the Nyquist plot as for transfer functions with real coefficients. However, some adjustments need to be made because the Nyquist plots of complex transfer functions for positive and negative frequencies are not necessarily the complex conjugates of each other.

To compute the delay margin, the compelex loop transfer function $GH(s)$ is obtained from (21) with (22),

$$\begin{aligned} GH(s) &= \frac{\boldsymbol{i}_g}{\boldsymbol{i}_g^{ref} - \boldsymbol{i}_g} \\ &= k_P \frac{v_{dc}\left(s + \frac{1}{T_i}\right)}{s\left(N_r(s) + v_{dc}k_f\left(1 + N_c(s)N_g(s)\right)\right)}, \end{aligned} \tag{25}$$

where (15)-(16) have been used. Figure 9 shows the Nyquist plot[3] of $GH(j\omega)$ for $-\infty < \omega < \infty$.

The phase margin is determined from the intersection of the Nyquist plot with the circle of radius 1 centered at (-1,0), and can be computed numerically. To address the differences between the real and complex transfer function cases, we define the phase margin as the angle $\phi_m \in (-\pi, \pi)$ such that

$$-e^{j\phi_m} = GH(j\omega_c), \tag{26}$$

where ω_c is the crossover frequency such that $|GH(j\omega_c)| = 1$. In this manner, ϕ_m can be positive or negative (for phase delay or phase advance), and an interval is obtained for the phase margin using the intersections for positive and negative frequencies (denoted ϕ_m^+ and ϕ_m^-, respectively). Since a delay T_d corresponds to a phase delay of ωT_d, a phase margin ϕ_m corresponds to a delay margin,

$$T_d = \frac{\phi_m}{\omega_c}. \tag{27}$$

[3]The Matlab® `nyquist` command does not work for complex transfer functions because the function only computes the polar plot for positives frequencies (and then takes the symmetric curve for the negatives ones).

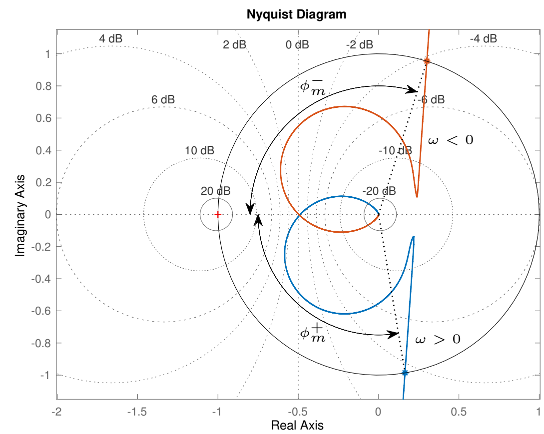

Fig. 9. Nyquist plot of $GH(j\omega)$ for $\omega > 0$ (in blue) and $\omega < 0$ (in red) using the parameters from Section V. The phase margins for the positive and negative frequencies are $\phi_m^+ = 1.736$rad and $\phi_m^- = 1.876$rad, respectively, with their corresponding crossover frequencies at $\omega_c^+ = 256.8$rad/s and $\omega_c^- = -257.2$rad/s.

Applying these concepts to the system with the parameters from Section V, we get that the phase margins are $\phi_m^+ = 1.736$rad and $\phi_m^- = -1.876$rad, with crossover frequencies $\omega_c^+ = 256.8$rad/s and $\omega_c^- = -257.2$rad/s. This corresponds to delay margins $T_d^+ = 6.7$ms and $T_d^- = 7.3$ms. Therefore, an overall delay margin of 6.7ms is obtained. The delay is far greater than the delay associated with sampling (0.05ms for 20kHz, see Section V), or with the current filter (less than $1\mu s$ for a DSP running at 160MHz). Additionally, the gain margins are satisfactory, $g_m^+ = 5.96$dB and $g_m^- = 5.81$dB, see Figure 9, and giving an overall gain margin of 5.81dB. The same analysis for the negative feedback control loop (24) gives even higher delay margins, $T_d^+ = 76.3$ms, and $T_d^- = 83.3$ms.

V. Simulation and experimental results

The proposed control strategy has been validated through simulation and experimental results. The simulation tests were performed using a complete model of the system which includes losses in the power converter and switching effects. Moreover, the experimental results were obtained using an LCL inverter laboratory prototype. The parameters used in simulation match the experimental system, and are: $v_{dc} = 300$V, $v_{ll} = 175$V, $L_f = 1.25$mH, $L_g = 0.625$mH, $R_f = 0.2\omega_g$, $R_g = 0.2\omega_g$ and $C = 4.4\mu$F, with a resistor of $R_c = 100$kΩ in parallel to damp the resonance of the LCL filter.

For a practical implementation, where a switched action is required, a frequency analysis is necessary to avoid resonance phenomena due to the switching frequency. From (17) we can deduce,

$$i_g = \frac{v_{dc}(N_r(s) - jN_i(s))}{N_r^2(s) + N_i^2(s)}u = \big(G_r(s) + jG_i(s)\big)u,$$

where,

$$G_r(s) = \frac{v_{dc}N_r(s)}{N_r^2(s) + N_i^2(s)}, \quad G_i(s) = \frac{v_{dc}N_i(s)}{N_r^2(s) + N_i^2(s)}.$$

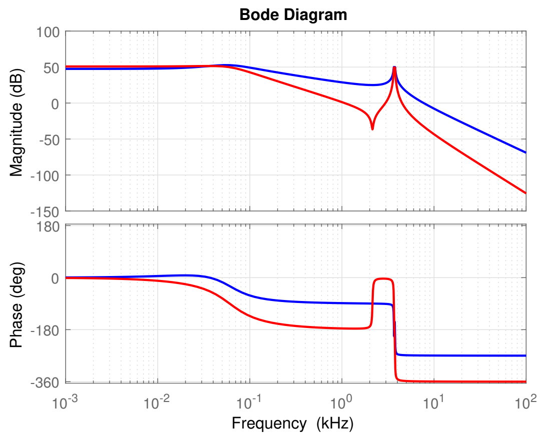

Fig. 10. Bode plots of $G_r(s)$ and $G_i(s)$.

Figure 10 shows the Bode plots of $G_r(s)$ (blue) and $G_i(s)$ (red). Notice that a first resonance peak appears close to 50Hz, a pair of resonance peaks are around 3.5kHz. This suggests commutation at higher frequencies, and the switching frequency of the PWM strategy was set at 20kHz.

A. Simulation results

With the aim of considering a realistic scenario in the simulation tests, the grid voltage has 2% of a fifth harmonic, 1% of a seventh harmonic and 10% of unbalance (calculated according to IEC standard). In addition, parameter variations in the LCL filter were considered (10% in L_f, L_g, R_f and R_g). The simulated test consist in three different scenarios:

A) From $t = 1$s, only the positive sequence is regulated at a current reference of 1.225A and unity power factor, corresponding to[4] $i_{gd}^{\text{ref}} = 1.5$A and $i_{gq}^{\text{ref}} = 0$A.
B) From $t = 1.25$s, control of the negative sequence controller, *i.e.,* the proposed negative sequence control (24) is turned on, with $i_{gd}^{\text{ref}-} = i_{gq}^{\text{ref}-} = 0$A.
C) From $t = 1.75$s, a step change $i_{gd}^{\text{ref}} = 2$A and $i_{gq}^{\text{ref}} = 0$.

Figures 11 and 12 show the three-phase currents and the positive and negative sequences of the grid currents during the test, respectively. Also, Figure 11 includes a zoom of the transient between scenarios from A to B and from B to C. It can be noticed how, initially, the proposed control scheme regulates only the positive sequence of the grid currents, i_g, at the desired values but, as expected, they are unbalanced. From $t = 1.25$s, the grid currents become balanced with the designed settling time, 0.2s. Then, at $t = 1.75$s, the reference of d axis grid current, i_{gd}^{ref}, changes from 1.5A to 2A, while $i_{gq}^{\text{ref}} = 0$. The control response can be observed in Figures 11 and 12, and exhibits a time response with less than one grid period and small overshoot, as expected.

B. Experimental tests

The experimental tests were performed using an LCL three-phase grid inverter laboratory prototype with the estimated parameters used during the simulation stage. The inverter has a semix-101GD12E4s module of Semikron (1200V, 100A) and the controller algorithm was implemented in a TMS320F28335 floating point DSP of Texas Instrument. Figure 13 shows the test-bench, including the inverter and the micro-controller. All tests in this section were performed using a real grid voltage with 2.63% of total harmonic distortion (THD) and 6% of unbalance calculated according IEC standard. The tests that were carried out are the same as the simulation tests presented in the previous subsection with scenarios A, B and C.

[4]The difference between the maximum value of i_{ga} and i_{gd} is because in the implementation of the controller, the power invariant abc/dq transformation was used, see (5).

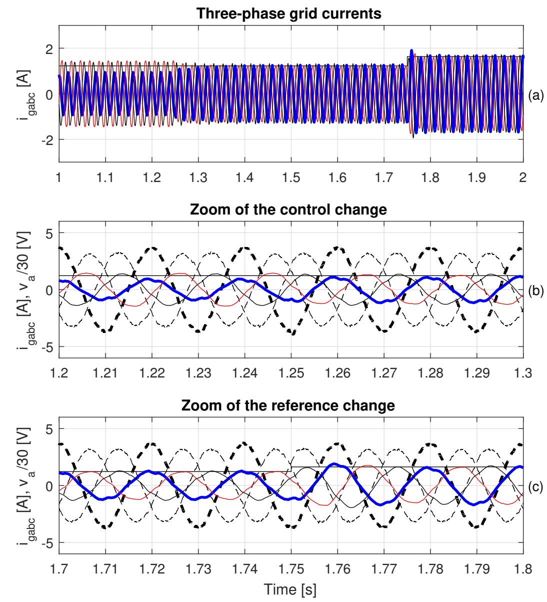

Fig. 11. Simulation results: Grid currents for a grid current reference change and negative sequence control. (a) Grid current in *abc* coordinates, (b) voltage and current of the system when activating the control of negative sequence, (c) voltage and current of the system for a change on the current reference.

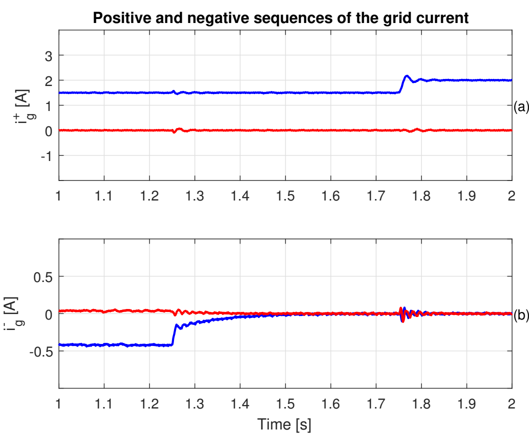

Fig. 12. Simulation results: Positive and negative sequences of the grid currents in dq coordinates. (a) i_{gd} and i_{gq}, (b) i_{gd}^- and i_{gq}^-.

Figure 14 shows the grid currents, for both positive (a) and negative (b) sequences, during the test. It can be observed that, between 1s and 1.25s the controller regulates the dq-currents, i_{gd} and i_{gq}, with the desired performance. Nevertheless, the grid currents are unbalanced because the control of the nega-

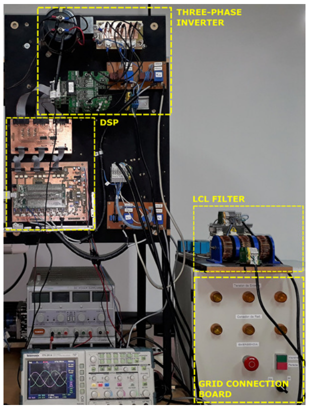

Fig. 13. Experimental testbench.

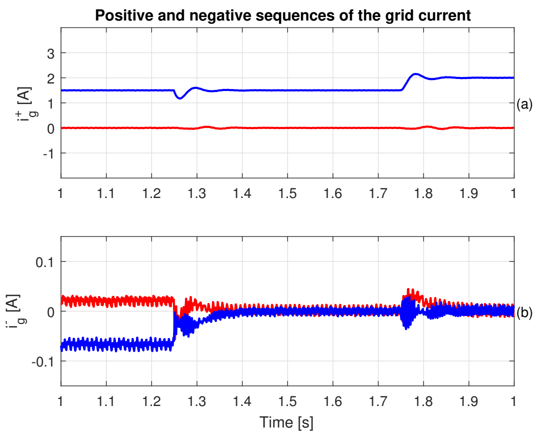

Fig. 14. Experimental results: Positive and negative sequences of the grid currents in dq coordinates. (a) i_{gd} and i_{gq}, (b) i_{gd}^- and i_{gq}^-.

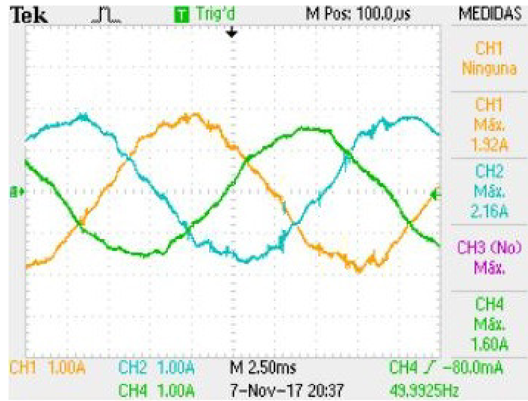

Fig. 15. Experimental results: Three-phase grid current in steady-state when the control of the negative sequence is off.

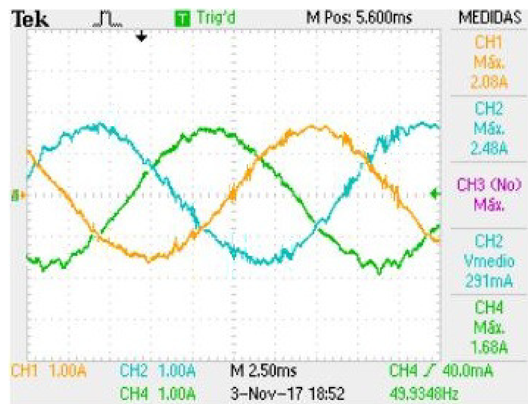

Fig. 16. Experimental results: Three-phase grid current in steady-state when the control of the negative sequence is turned on.

tive sequence is not used. From $t = 1.25$s, when the control of the negative sequence is turned on, the grid currents become balanced in the specified time. In the same test, at $t = 1.75$s, the reference of d axis grid current, i_{gd}^{ref}, changes from 1.5A to 2A, while $i_{gq}^{\text{ref}} = 0$. The control response is as expected with approximately one grid period and with small overshoot.

Figures 15 and 16 plot the steady-state currents before and after the control of the negative sequence current is engaged. As expected, the currents are balanced, even though the grid voltages are unbalanced, when the negative sequence control is on.

The behavior of the three-phase grid currents for a change of the reference current (from $i_{gd}^{\text{ref}} = 1.5$A to $i_{gd}^{\text{ref}} = 2$A) is represented in Figure 17. As can be observed, the time response of the currents to the step change corresponds to the desired value, one grid period.

Finally, Figure 18 shows a detail of the grid voltage and current for phase a when $i_{gd}^{\text{ref}} = 2$A and $i_{gq}^{\text{ref}} = 0$A (unity power factor). In steady state, the grid current has low distortion and is in phase with the voltage. As shown in Figure 19, even though the controller only considers the fundamental (with both positive and negative sequences), the THD of the current is 2.76% (within the standard values, that is below 5%), that mainly appear because the THD of the voltage grid.

VI. Conclusions

This paper uses a new approach for the design of controllers in electrical power systems based on the description of the dynamics using transfer functions with complex coefficients. The methodology has been applied to a three-phase grid inverter with an LCL filter, including practical scenarios such as parameter variations, unbalanced grids and the effect of the delays. The use of the complex description of the plant allows one to explore alternatives to the approaches reported in the literature. In particular, the use of SISO tools for a MIMO problem implies that the typical decoupling term is not necessary. In fact, the resulting design suggests a coupling of the dq coordinates by means of a complex value of k_f, moving the starting poles of the root locus to the left and improving the stability margin. The design procedure was detailed, including discussions on the root locus. Both simulations and experimental results validate the controller design, and show the usefulness of the proposed gain tuning procedure for achieving the desired performance. Future works include the use of the presented approach for systems with high THD, as well as its application to other electrical power systems.

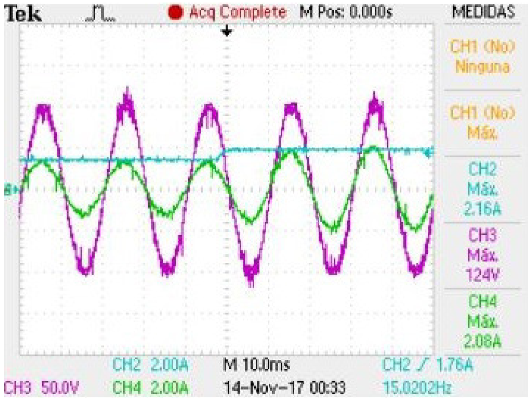

Fig. 17. Experimental results: Grid voltage (blue) and transient of the grid current (red) for a step change in i_{gd}^{ref} from 1.5A to 2A (magenta).

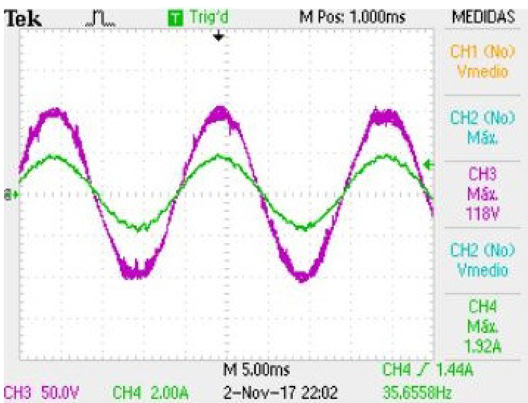

Fig. 18. Experimental results: Grid voltage (blue) and steady-state grid current (red) for $i_{gd}^{\text{ref}} = 2.5$A.

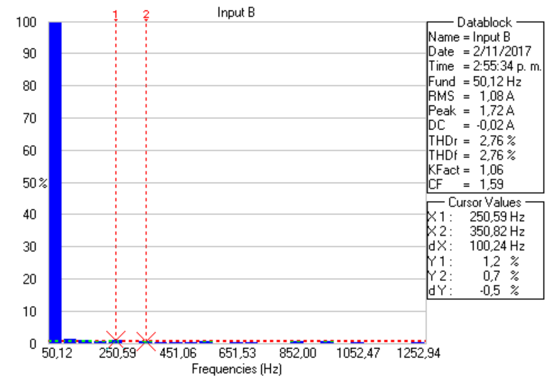

Fig. 19. Experimental results: Frequency spectrum corresponding to the grid current shown in Figure 18.

References

[1] R. Teodorescu, M. Liserre, and P. Rodríguez, *Grid converters for photovoltaic and wind power systems*. John Wiley & Sons, 2011.

[2] M. Castilla, J. Miret, A. Camacho, J. Matas, and L. de Vicuña, "Reduction of current harmonic distortion in three-phase grid-connected photovoltaic inverters via resonant current control," *IEEE Trans. on Industrial Electronics*, vol. 60, no. 4, pp. 1464–1473, 2013.

[3] X. Zhang, P. Chen, C. Yu, F. Li, H. Thanh-Do, , and R. Cao, "Study of a current control strategy based on multisampling for high-power grid-connected inverters with an LCL filter," *IEEE Trans. on Power Electronics*, vol. 32, no. 7, pp. 5023–5034, 2017.

[4] D. Pan, X. Ruan, C. Bao, W. Li, and X. Wang, "Optimized controller design for LCL-type grid-connected inverter to achieve high robustness against grid-impedance variation," *IEEE Trans. on Industrial Electronics*, vol. 62, no. 3, pp. 1537–1547, 2015.

[5] R. Fantino, C. Busada, and J. Solsona, "Optimum PR control applied to LCL filters with low resonance frequency," *IEEE Trans. on Power Electronics*, vol. 33, no. 1, pp. 793–801, 2018.

[6] H. Komurcugil, N. Altin, S. Ozdemir, and I. Sefa, "Lyapunov-function and proportional-resonant-based control strategy for single-phase grid-connected VSI with LCL filter," *IEEE Trans. on Industrial Electronics*, vol. 63, no. 5, pp. 2838–2849, 2016.

[7] C. Busada, S. Gomez-Jorge, and J. Solsona, "Full-state feedback equivalent controller for active damping in LCL-filtered grid-connected inverters using a reduced number of sensors," *IEEE Trans. on Industrial Electronics*, vol. 62, no. 10, pp. 5993–6002, 2015.

[8] S. Khajehoddin, M. Karimi-Ghartemani, and M. Ebrahimi, "Optimal and systematic design of current controller for grid-connected inverters," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Early access article.

[9] F. Huerta, J. Pérez, S. Cóbreces, and M. Rizo, "Frequency-adaptive multi-resonant LQG state-feedback current controller for LCL- filtered VSCs under distorted grid voltages," *IEEE Trans. on Industrial Electronics*, Early Access, 2018.

[10] X. Zhang, Y. Wang, C. Yu, L. Guo, and R. Cao, "Hysteresis Model Predictive Control for high-power grid-connected inverters with output LCL filter," *IEEE Trans. on Industrial Electronics*, vol. 63, no. 1, pp. 246–256, 2016.

[11] B. Wang, Y. Xu, Z. Shen, J. Zou, C. Li, and H. Liu, "Current control of grid-connected inverter with LCL filter based on extended-state observer estimations using single sensor and achieving improved robust observation dynamics," *IEEE Trans. on Industrial Electronics*, vol. 64, no. 7, pp. 5428–5439, 2017.

[12] J. Wang, J. Yan, and L. Jiang, "Pseudo-derivative-feedback current control for three-phase grid-connected inverters with LCL filters," *IEEE Trans. on Power Electronics*, vol. 31, no. 5, pp. 3898–3912, 2016.

[13] N. He, D. Xu, Y. Zhu, J. Zhang, G. Shen, Y. Zhang, J. Ma, and C. Liu, "Weighted average current control in a three-phase grid inverter with an LCL filter," *IEEE Trans. on Power Electronics*, vol. 28, no. 6, pp. 2785–2797, 2013.

[14] A. Dòria-Cerezo and M. Bodson, "Design of controllers for electrical power systems using a complex root locus method," *IEEE Trans. on Industrial Electronics*, vol. 63, no. 6, pp. 3706–3716, 2016.

[15] Y. Han, Z. Li, P. Yang, C. Wang, L. Xu, and J. Guerrero, "Analysis and design of improved weighted average current control strategy for LCL-type grid-connected inverters," *IEEE Trans. on Energy Conversion*, vol. 32, no. 3, pp. 941–952, 2017.

[16] D. Biel, A. Dòria-Cerezo, V. Repecho, and E. Fossas, "Sliding mode control of a three-phase three-wire LCL rectifier," in *Proc. 39th Annual Conf. of the IEEE Industrial Electronics Society*, 2014.

[17] R. Guzman, L. G. de Vicuña, M. Castilla, J. Miret, and J. de la Hoz, "Variable structure control for three-phase LCL-filtered inverters using a reduced converter model," *IEEE Trans. on Industrial Electronics*, vol. 65, no. 1, pp. 5–15, 2018.

[18] S. Eren, M. Pahlevaninezhad, A. Bakhshai, and P. Jain, "Composite nonlinear feedback control and stability analysis of a grid-connected voltage source inverter with LCL filter," *IEEE Trans. on Industrial Electronics*, vol. 60, no. 11, pp. 5059–5074, 2013.

[19] Y. He, H.-H. Chung, C.-M. Ho, and W. Wu, "Modified cascaded boundary-deadbeat control for a virtually-grounded three-phase grid-connected inverter with LCL filter," *IEEE Trans. on Power Electronics*, vol. 32, no. 10, pp. 8163–8180, 2017.

[20] X. Guo, W. Liu, X. Zhang, X. Sun, Z. Lu, and J.-M. Guerrero, "Flexible control strategy for grid-connected inverter under unbalanced grid faults without PLL," *IEEE Trans. on Power Electronics*, vol. 30, no. 4, pp. 1773–1778, 2015.

[21] X. Guo, W. Liu, and Z. Lu, "Flexible power regulation and current-limited control of the grid-connected inverter under unbalanced grid voltage faults," *IEEE Trans. on Industrial Electronics*, vol. 64, no. 9, pp. 7425–7432, 2017.

[22] Y.-R. Mohamed, "Mitigation of dynamic, unbalanced, and harmonic voltage disturbances using grid-connected inverters with LCL filter," *IEEE Trans. on Industrial Electronics*, vol. 58, no. 9, pp. 3914–3924, 2011.

[23] Z. Xin, P. Mattavelli, W. Yao, Y. Yang, F. Blaabjerg, and P. Loh, "Mitigation of grid current distortion for LCL-filtered voltage source

inverter with inverter current feedback control," *IEEE Trans. on Power Electronics*, Early access article.

[24] N.-B. Lai and K.-H. Kim, "Robust control scheme for three-phase grid-connected inverters with LCL-filter under unbalanced and distorted grid conditions," *IEEE Trans. on Energy Conversion*, Early Access, 2018.

[25] D. Pérez-Estévez, J. Doval-Gandoy, A. Yepes, and O. López, "Positive- and negative-sequence current controller with direct discrete-time pole placement for grid-tied converters with LCL filter," *IEEE Trans. on Power Electronics*, vol. 32, no. 9, pp. 7207–7221, 2017.

[26] D. Novotny and J. H. Wouterse, "Induction machine transfer functions and dynamic response by means of complex time variables," *IEEE Trans. on Power Apparatus and Systems*, vol. 95, no. 4, pp. 1325–1335, 1976.

[27] P. Dalton and V. Gosbell, "A study of induction motor current control using the complex number representation," in *Proc. IEEE Industry Applications Society Annual Meeting*, 1989.

[28] S. Gataric and N. Garrigan, "Modeling and design of three-phase systems using complex transfer functions," in *Proc. 30th Annual IEEE Power Electronics Specialists Conference*, 1999.

[29] F. B. del Blanco, M. Degner, and R. Lorenz, "Dynamic analysis of current regulators for AC motors using complex vectors," *IEEE Trans. on Industry Applications*, vol. 35, no. 5, pp. 1424–1423, 1999.

[30] L. Harnefors, "Modeling of three-phase dynamic systems using complex transfer functions and transfer matrices," *IEEE Trans. on Industrial Electronics*, vol. 54, no. 4, pp. 2239–2248, 2007.

[31] M. Bodson, "The complex Hurwitz test for the stability analysis of induction generators," in *Proc. 2010 American Control Conference*, 2010.

[32] M. Bodson and O. Kiselychnyk, "The complex Hurwitz test for the analysis of spontaneous self-excitation in induction generators," *IEEE Trans. on Automatic Control*, vol. 58, no. 2, pp. 449–454, 2013.

[33] A. Dòria-Cerezo, M. Bodson, C. Batlle, and R. Ortega, "Study of the stability of a direct stator current controller for a doubly-fed induction machine using the complex Hurwitz test," *IEEE Trans. on Control Systems Technology*, vol. 21, no. 6, pp. 2323–2331, 2013.

[34] M. Bodson, "Design of controllers in the complex domain," in *Proc. 53th Conference on Decision and Control*, 2014.

[35] O. Troeng, B. Bernhardsson, and C. Rivetta, "Complex-coefficient systems in control," in *Proc. 2017 American Control Conference*, 2017.

[36] X. Guo, W. Wu, and Z. Chen, "Multiple-complex coefficient-filter-based phase-locked loop and synchronization technique for three-phase grid-interfaced converters in distributed utility networks," *IEEE Trans. on Industrial Electronics*, vol. 58, no. 4, pp. 1194–1204, 2011.

[37] A. Dòria-Cerezo and M. Bodson, "Root locus rules for polynomials with complex coefficients," in *Proc. 21st Mediterranean Conference on Control and Automation*, 2013.

[38] J. Dannehl, F. Fuchs, and P. Thogersen, "PI state space current control of grid-connected PWM converters with LCL filters," *IEEE Trans. on Power Electronics*, vol. 25, no. 9, pp. 2320–2330, 2010.

[39] A. Dabroom and H. Khalil, "Numerical differentiation usign high-gain observers," in *Proc. 36th Conference on Decision and Control*, 1997.

[40] A. Yazdani and R. Iravani, "A unified dynamic model and control for the voltagesourced converter under unbalanced grid conditions," *IEEE Trans. on Power Delivery*, vol. 21, no. 3, pp. 1620–1629, 2006.

[41] P. Rodríguez, A. Luna, I. Candela, R. Mujal, R. Teodorescu, and F. Blaabjerg, "Multiresonant frequency-locked loop for grid synchronization of power converters under distorted grid conditions," *IEEE Trans. on Industrial Electronics*, vol. 58, no. 1, pp. 127–138, 2011.

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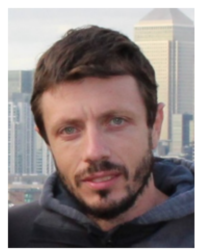

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